

# Second Order Thermal Corrections to Electron Wavefunction

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## Abstract

Second order perturbative corrections to electron wavefunction are calculated here at generalized temperature, for the first time. This calculation is important to prove the renormalizability of QED through order by order cancellation of singularities at higher order. This renormalized wavefunction could be used to calculate the particle processes in the extremely hot systems such as the very early universe and the stellar cores. We have to re-write the second order thermal correction to electron mass in a convenient way to be able to calculate the wavefunction renormalization constant. A procedure for integrations of hot loop momenta before the cold loop momenta integration is maintained throughout to be able to remove hot singularities in an appropriate way. Our results, not only includes the intermediate temperatures  $T \sim m$  (where  $m$  is the electron mass), the limits of high temperature  $T \gg m$  and low temperature  $T \ll m$  are also retrievable. A comparison is also done with the existing results.

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## 1 Introduction

Finite temperature effects are important in extremely high temperature environments, such as in the early Universe a few seconds after the Big Bang, astrophysical environments etc., where they are significant enough and can not be ignored in comparison with the vacuum contribution. The high temperature and density effects in ultra-relativistic plasma need to be incorporated, for example, in Quantum Electrodynamics (QED) plasma, quark gluon plasma and the core of dense stars. More recently renewed interest in hot and dense QED plasmas has been generated due to the possibility of creating ultra-relativistic electron positron plasmas with high-intensity lasers ( $\approx 10^{18} \text{W/cm}^2$ ) [1-3]. Two opposite laser pulses hitting a thin gold foil can heat up the electrons in the foil up to several MeV ( $\sim 10^{10} \text{K}$ ).

The particles propagating in vacuum can be assumed to be the ones with interactions switched off. When these particles propagate through a medium, several kinds of interaction processes take place. This makes the properties of the system different from that in which all the particles are assumed to be completely independent of each other, behaving as freely propagating bare particles. When dealing with finite temperature environments in QED, where the particles propagate in statistical background at energies around the thresholds for

particle antiparticle pair production, the temperature effects need to be appropriately taken into account. These effects arise due to continuous electron and photon exchanges between particles during the physical interactions that take place in a heat bath containing hot particles and antiparticles. The net statistical effects of the background electrons and photons enter the theory through the fermion and boson distributions respectively. Finite temperature calculations also provide a guideline to estimate the density corrections, through chemical potential effects of the background plasma, for higher order loop corrections.

The thermal background effects are included through the radiative corrections [4,5]. Self energies and the wavefunctions of the propagating particles acquire temperature corrections in this environment due to exchanges of energy and momentum with real particles. The exact state of all these background particles is unknown since they continually fluctuate between different configurations. Temperatures of interest in such a situation are in the range of a few MeV. Thermal propagators in real time formalism comprise of temperature dependent terms added to the particle propagators in vacuum theory [6]. In finite temperature electrodynamics, electric fields are screened due to such interactions.

We use the real time formulation [7] for calculations of wavefunction renormalization as a second order perturbative correction in  $\alpha$  due to the ease of obtaining the temperature corrections as additive terms to the usual contribution in vacuum. Here we prefer that the loops with temperature dependent momenta are integrated before temperature independent variables in the relevant order  $\alpha^2$  loops in QED, and therefore, review the electron self energy in Ref. [8,9] earlier. This makes the calculations of integrations over loop momenta much more simpler and easier to handle. The results are obtained in a generalized form such that intermediate temperatures  $T \sim m$  are also included while the ranges of high temperature  $T \gg m$  and low temperature  $T \ll m$ , are retrieved from them as the limiting cases.

In literature, the ways to compute finite temperature effects on phase-space, vertex, mass corrections and photon emission or absorption [10-19] are extensively discussed. The finite temperature wave function renormalization has been dealt with several approaches [12-21], specifically in the context of weak decay rates during primordial nucleosynthesis. They agree on using finite temperature Dirac spinors to obtain the corresponding effective projection operator. Differences in the spinors presented in Refs. [20] and [21] were also pointed out [22]. However, their results for the case of  $\beta$ -decay and related processes agreed with the ones which can be obtained using the approaches that had already existed for wave function renormalization, except for the case of a scalar boson decay in fermion-antifermion pair. We work out the two loop corrections to the finite temperature wave function renormalization in the generalized temperature framework, for the first time. Section 2 is based on the re-examined and simplified calculations of loop correction upto two orders in  $\alpha$  that contribute to electron self energy in this background. The expression for the relative correction in electron mass is redone and the wave function renormalization constant is calculated in section 3. Section 4 gives discussion of the results.

## 2 Loop corrections to electron self energy

At the one loop level, Feynman diagrams are calculated in the usual way by substituting the finite temperature electron, positron and photon propagators in place of those in vacuum. In real time formalism, the finite temperature terms remain separate at order  $\alpha$  since the terms depending on temperature (hot) are additive to temperature independent (cold) terms in the propagators. Therefore, at the one loop level the hot and cold loop momenta are integrated separately.

Due to interactions with the background, the electron and positron masses are known to get enhanced at one-loop and higher loop levels [4-9]. The photons also acquire dynamically generated mass due to plasma screening effect [23, 24, 27]. The presence of effective mass implies the fact that the propagating particles constantly interact with the background. The radiatively generated thermal mass creates a mass shift and in physical quantities this acts as a kinematical cut-off, e.g., while determining the production rate of particles in the heat bath.

Higher order loop corrections are required to study the perturbative behavior at finite temperature. The two loop integrals comprise a combination of cold and hot momenta which appear due to an overlap of temperature dependent and temperature independent terms from the particle propagators. The loop integrations involve an overlap of finite and divergent terms due to which these become analytically more complicated. In such situations, a preferred approach needs to be adopted for integrating overlapping hot and cold loop momenta, i.e., to integrate over hot loop momenta before integrating over cold ones, even at the two loop level [24]. This not only helps to simplify the loop integrations but allows one to handle the statistical effects more appropriately.

The problem of renormalization in finite temperature field theories is somewhat different from that at zero temperature due to the additional hot infrared divergences at finite temperatures. The temperature, however, acts as a regularization parameter for the hot ultraviolet divergences. The infrared divergences introduced in this framework are also appropriately removable in particle decay processes via bremsstrahlung emission and absorption effects [5, 25]. This was studied in detail at order  $\alpha$  for all the possible ranges of temperature valid in QED including  $T \sim m$  [25, 27]. The renormalization of QED was also established at the one loop level for all the relevant ranges in temperatures and chemical potential [23, 25-27].

Fig. 1. Two loop electron self energy diagrams and counter terms

The electron self energy is once again calculated here at the two loop level, from

the point of view of renormalization, by integrating over loop momenta in the electron self energy diagrams in Fig. 1(a) and 1(b). The relevant counter terms required to cancel the divergences in these two loop self energies are included in Fig. 1(c) and 1(d). Removal of overlapping divergences has been even checked in QED up to the two loop level for  $T \sim m$ ,  $T \ll m$  and  $T \gg m$  [9]. The integrations over the temperature dependent momenta are re-examined here and wherever needed are re-done before temperature independent momentum variables in loops for all the ranges of temperature that are relevant in QED. The overlapping loops in Fig.1(a) gives nonzero real contribution to be

$$\begin{aligned}
\Sigma^a(p) = & \frac{\alpha^2}{2} \left[ \frac{1}{2\pi^3} \left\{ \frac{1}{\epsilon} [2I - (\not{p} + 6m)I_A + 3(2m - \not{p})J_A + 2J_B] - 3(\not{p} + 4m)I_A \right. \right. \\
& + 4I + (12m - 5\not{p})J_A + 2J_B \} + (-1)^{r+1} e^{-r\beta E} \sum_{n,r,s=1}^{\infty} \{ [\frac{3T^2}{4} \{ f_+(n,r) \\
& \times [mf_+(s,r)(\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2})^2 - f_-(s,r)h_-(p,\gamma)\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2}] - f_-(n,r)[f_+(s,r)h_-(p,\gamma)\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2} \\
& - \frac{1}{m} \{ 1 - \frac{2}{3}\gamma^0 \frac{\gamma \cdot \mathbf{P}}{v|\mathbf{p}|} + (\frac{\gamma \cdot \mathbf{P}}{v|\mathbf{p}|})^2 \} f_-(s,r) \} + 2T[(4 + \not{p}\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2})f_+(s,r) \\
& + \not{p}h_-(p,\gamma)\{f_-(s,r) - f_-(n,r)\frac{1}{m}\frac{I_C}{8\pi}\} + \frac{I_B}{8\pi}\{(4 - \not{p}\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2})f_+(n,r) \\
& + \frac{(\not{p} + m)I_C}{8\pi}\}] \} + (-1)^s [T^2 \{ f_+(n,r)[\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2}(1 - 3m\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2})f_+(s,r) \\
& + 3h_-(p,\gamma)\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2}f_-(s,r)] - [\frac{1}{m}h_-(p,\gamma) - 3m(\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2})^2\}f_+(s,r) \\
& - \frac{3}{m}h_+(p,\gamma)f_-(s,r)]f_-(n,r) \} - T\{[(5\not{p} + 3m^2\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2})\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2} - 12\}f_+(n,r) \\
& + \frac{5\not{p}}{m}h_-(p,\gamma)f_-(n,r)]\text{Ei}_- - [\frac{1}{m}f_-(n,r)h_+(p,\gamma) - \frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2}f_+(n,r)h_-(p,\gamma)] \\
& \times 3E\text{Ei}_+ + \{[3h_-(p,\gamma)\frac{\gamma \cdot \mathbf{P}}{v^2}f_+(n,r) - \frac{1}{m}f_-(n,r)\{3|\mathbf{p}|^2h_+(p,\gamma) \\
& - 5E\not{p}h_-(p,\gamma)\}][\frac{2e^{-rm\beta}}{m}\sinh sm\beta + \frac{(r\text{Ei}_+ + s\text{Ei}_-)}{T}] - [\frac{(r\text{Ei}_+ - s\text{Ei}_-)}{T} \\
& + \frac{2e^{-rm\beta}}{m}\cosh sm\beta]m[\{h_-(p,\gamma) - 3m(\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2})^2\}f_-(n,r) - \frac{m}{2}\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2} \\
& \times (1 - 3m\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2})f_+(n,r)] - [2\gamma^0 T\{1 + \frac{T}{m}f_+(s,r)\} + \not{p}T\{\frac{2}{m}h_-(p,\gamma) \\
& - \frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2}\}f_-(s,r) + \{E\text{Ei}_+ - m\gamma^0\text{Ei}_-\} + \not{p}\{\frac{m^2}{2}\frac{\gamma \cdot \mathbf{P}}{|\mathbf{p}|^2} + \frac{2E^2}{m}h_-(p,\gamma)\} \\
& \times \{ \frac{e^{-m\beta(s+r)} - e^{-m\beta(r-s)}}{m} + \beta(r\text{Ei}_+ - s\text{Ei}_-)\} \} \frac{I_C}{8\pi} \} \}. \tag{1}
\end{aligned}$$

where

$$f_{\pm}(n, r) = \left\{ \frac{1}{(n+r)} \pm \frac{1}{(n-r)} \right\}; \quad f_{\pm}(s, r) = \left\{ \frac{1}{(s+r)} \pm \frac{1}{(s-r)} \right\},$$

$$h_{\pm}(p, \gamma) = (\gamma^0 \pm \frac{\boldsymbol{\gamma} \cdot \mathbf{p}}{v|\mathbf{p}|}),$$

$$\text{Ei}_{\pm} = \text{Ei}[-m\beta(r+s)] \pm \text{Ei}[-m\beta(r-s)],$$

$$I_A = 8\pi \int_0^{\infty} \frac{dk}{k} n_B(k),$$

$$I_B = 8\pi \sum_{r=1}^{\infty} (-1)^r \int_0^{\infty} \frac{dk}{k} e^{-r\beta(p-k)} n_B(k),$$

$$I_C = 8\pi \sum_{r=1}^{\infty} (-1)^r \int_0^{\infty} \frac{dk}{k} e^{-r\beta k} n_B(k),$$

$$J^A \simeq -8\pi b(m\beta),$$

$$\frac{I^0}{E} = -\frac{2\pi^3 T^2}{3E^2 v} \ln \frac{1-v}{1+v},$$

$$\frac{\mathbf{I} \cdot \mathbf{p}}{\mathbf{p}^2} = -\frac{2\pi^3 T^2}{3E^2 v^3} \left\{ \ln \frac{1-v}{1+v} + 2v \right\},$$

$$\frac{J_B^0}{E} \simeq 4\pi \left[ \frac{T}{\mathbf{p}E} \ln \frac{1+v}{1-v} \{ma(m\beta) - Tc(m\beta)\} - 3b(m\beta) \right],$$

$$\frac{\mathbf{J}_B \cdot \mathbf{p}}{\mathbf{p}^2} \simeq \frac{\pi}{v^2 E^2} \left[ \{E^2 - \frac{2}{3}m^2\} b(m\beta) + 4T \left\{ \frac{1}{v} \ln \frac{1+v}{1-v} + 2 \right\} \{ma(m\beta) - Tc(m\beta)\} \right],$$

with  $v = \frac{|\mathbf{p}|}{p_0}$ , ( $p_0 = E$ ),

$$a(m\beta) = \ln(1 + e^{-m\beta}),$$

$$b(m\beta) = \sum_{n=1}^{\infty} (-1)^n \text{Ei}(-nm\beta),$$

$$c(m\beta) = \sum_{n=1}^{\infty} (-1)^n \frac{e^{-nm\beta}}{n^2},$$

and  $\text{Ei}(-x)$  is the error integral given by

$$\text{Ei}(-x) = - \int_x^{\infty} \frac{dt}{t} e^{-t}.$$

The non vanishing real terms from loop within loop correction in Fig.1(b) are also re-examined and recalculated, wherever needed, by retaining the specific

order of integrating loop momenta, i.e., on the integration over the variables in hot momenta before the cold momenta, we get

$$\begin{aligned}
\Sigma_\beta^b(p) = & \alpha^2 \left\{ \frac{2T^2}{3m^2} (\not{p} - m) + mT \sum_{n,r,s=1}^{\infty} (-1)^{s+r} [\{\beta(r+s) \text{Ei}[-m\beta(r+s)] \right. \\
& + \frac{e^{-m\beta(s+r)}}{m} \} \{ \frac{1}{(n-r)} [\frac{2E}{m} + \gamma^0 (\frac{1}{2} - \frac{E^2}{m^2}) + r\beta(m - \frac{\not{p}}{2})] \\
& + \frac{1}{(n-s)} [h(p, \gamma) + m \frac{\gamma \cdot \mathbf{p}}{|\mathbf{p}|^2}] \} + \frac{1}{2(n-r)} \{ \frac{2}{m} - \frac{E\gamma^0}{m^2} \\
& + r\beta[h(p, \gamma) + m \frac{\gamma \cdot \mathbf{p}}{|\mathbf{p}|^2}] \text{Ei}[-m\beta(r+s)] \} \} \\
& - \frac{\pi T}{6|\mathbf{p}|} \sum_{n,r,s=1}^{\infty} (-1)^{r+1} e^{-n\beta E} [h(p, \gamma) \{ [1 + (-1)^s] \frac{e^{-m\beta(r-n-s)}}{r-n-s} \} \\
& - T \frac{\gamma \cdot \mathbf{p}}{|\mathbf{p}|^2} \frac{e^{-m\beta(r-n)}}{r-n} + (2 - \frac{m\gamma \cdot \mathbf{p}}{|\mathbf{p}|^2}) \{ m\beta(r-n) \text{Ei}[-m\beta(r-n)] \\
& - e^{-m\beta(r-n)} + \beta[1 + (-1)^{1+s}] \text{Ei}[-m\beta(r-n-s)] \} \} \}. \tag{2}
\end{aligned}$$

In Eqs. (1) and (2), the preferred order of integration not only sufficiently eases the calculations but the results are also simpler as compared to those in Ref. [9] where this preference was not realized. The electron self energies at the two loop level in Fig 1(a) and 1(b) are then combined and rearranged to obtain the temperature corrections to the electron mass and wavefunction.

### 3 The Wavefunction Renormalization

To incorporate finite temperature effect on physical processes beyond the tree level, one needs to have a consistent method of temperature dependent renormalization. As already mentioned, renormalizability of the electron mass was done through the order by order cancellation of singularities up to two loop level. It can be easily checked that the second order in  $\alpha$  correction is much smaller than the first order contribution so that the perturbative behavior is valid. In a background, with  $T \neq 0$ , the Lorentz invariance is broken and momentum independent renormalization constant is no longer sufficient. Donoghue and Holstein used the temperature dependent propagator to modify electron mass as well as the spinors accordingly [5].

The shift in the electron mass due to finite temperature effects is calculated here from Eqs. (1) and (2). For this all the finite terms in electron self energy upto second order in  $\alpha$  are put together. Following Ref. [5] the physical mass of the electron at one loop was obtained in Ref. [25] in generalized form, by writing

$$\Sigma(p) = A(p)E\gamma_0 - B(p)\vec{p} \cdot \vec{\gamma} - C(p),$$

where  $A(p)$ ,  $B(p)$ , and  $C(p)$  are the relevant coefficients. Taking the inverse of

the propagator with momentum and mass term separated as

$$S^{-1}(p) = (1 - A)E\gamma^o - (1 - B)p \cdot \gamma - (m - C),$$

and the physical mass  $m_{phy} = m + \delta m^{(1)} + \delta m^{(2)}$ , was deduced by locating the pole of the propagator  $\frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}$ .  $\delta m^{(1)}$  and  $\delta m^{(2)}$  is the shift in electron mass due to temperature effects at one and two loop level respectively. Using the same procedure, the relative shift in electron mass at the two loop level was obtained [9]. This is recalculated here, wherever required, and after recombining the similar summations it becomes:

$$\begin{aligned} \frac{\delta m^{(2)}}{m} = & 2\alpha^2 \sum_{r=1}^{\infty} \left[ \frac{T^2}{m^2} \left\{ \sum_{n=3}^{r+1} (-1)^{n+r+1} \frac{\pi m}{6|\mathbf{p}|} \frac{e^{-\beta(rE+mn)}}{n} \right. \right. \\ & - \frac{3}{8} (-1)^r \frac{e^{-r\beta E}}{|\mathbf{p}|^2} \left[ \frac{9E^2}{2m^2} + 6 \sum_{s=3}^{r+1} \frac{1}{s} + 4 \sum_{n,s=3}^{r+1} \frac{1}{ns} + (-1)^{s-r} \left\{ \frac{9E}{m} \left( 3 + 4 \sum_{s=3}^{r+1} \frac{1}{s} \right) \right. \right. \\ & + 2 \left( \frac{|\mathbf{p}|^2}{m^2} - 3 \right) \left( 9 + 18 \sum_{s=3}^{r+1} \frac{1}{s} + 8 \sum_{n,s=3}^{r+1} \frac{1}{ns} \right) \} \left. \right] + \frac{4}{E^2 v^2} \} - \frac{m^2}{\pi^2} c(m\beta) \\ & - \frac{T}{m} \left\{ \frac{\pi}{6|\mathbf{p}|} \sum_{s=2}^{r+1} \sum_{n=1}^{s+1} \frac{e^{-\beta(rE+mn)}}{n} [1 - \{(-1)^{r+n} - (-1)^{s+n}\}] \right. \\ & + [\{ \text{Ei}(-m\beta) - \text{Ei}(-2m\beta) \} \{ \frac{9E}{4} \left( \frac{E}{|\mathbf{p}|^2} - \frac{1}{m} \right) + \left( \frac{5E}{m} - 21 + \frac{E^2}{2m^2} \right) \sum_{n=3}^{r+1} \frac{1}{n} \} \\ & + \{ \frac{9}{4v^2} - \sum_{n=1}^{s+1} \sum_{s=3}^{r+1} [1 - E^2 \left( \frac{1}{2m^2} + \frac{3}{|\mathbf{p}|^2} \right) + \frac{3E}{m}] \} (-1)^s \text{Ei}(-sm\beta)] \\ & + e^{-rm\beta} \{ \left[ \frac{9E}{2v^2} + 2 \left( \frac{3E}{v^2} + \frac{3|\mathbf{p}|^2}{m} - 5E \right) \sum_{n=3}^{r+1} \frac{1}{n} \right] \sum_{s=1}^{\infty} \sinh sm\beta \\ & - \frac{3m^3}{|\mathbf{p}|^2} \left( \frac{3}{4} - \sum_{n=3}^{r+1} \frac{1}{n} \right) \sum_{s=1}^{\infty} \cosh sm\beta \} \} + \{ \frac{9m}{4|\mathbf{p}|^2} (E^3 + \frac{m^3}{2}) \\ & + [\frac{3m}{|\mathbf{p}|^2} (E^3 + m^3) + 5mE - 3|\mathbf{p}|^2] \sum_{n=3}^{r+1} \frac{1}{n} \} \{ \text{Ei}(-m\beta) - 2 \text{Ei}(-2m\beta) \} \\ & - \sum_{n=3}^{r+1} \{ \sum_{s=1}^{r+1} \frac{(-1)^s}{n} \left[ \frac{m^2 r}{2} e^{-sm\beta} + \{ s(2mE - \frac{E^3}{m}) + \frac{m^2(s-r)}{2} \} \text{Ei}(-sm\beta) \right] \\ & - \frac{\pi m^2}{3|\mathbf{p}|} [e^{-\beta rE} (-1)^{n+r} (n+1) - \sum_{s=2}^{r+1} (-1)^{n+s} \text{Ei}(-nm\beta)] \}. \end{aligned} \quad (3)$$

From the reviewed expression for the electron self energy obtained in Eqs. (1) and (2) the relation for the wave function renormalization constant is derived.

This comes out to be

$$\begin{aligned}
Z_2^{-1} &= \frac{\partial \Sigma}{\partial \not{p}} \\
&= 1 - \alpha \left[ \frac{1}{4\pi} \left( \frac{3}{\varepsilon} - 4 \right) + \frac{5}{\pi} b(m\beta) + \frac{I_A}{4\pi^2} \right. \\
&\quad - \frac{T^2}{\pi v E^2} \ln \frac{1+v}{1-v} \left\{ \frac{\pi^2}{6} - c(m\beta) + m\beta a(m\beta) \right\} \\
&\quad - \alpha^2 \left[ \frac{1}{4\pi^3} \left\{ \frac{1}{\varepsilon} (I_A - 3J_A) + (3I_A + 5J_A) \right\} - \frac{2T^2}{3\pi^3 m^2} \right. \\
&\quad + \frac{m}{8\pi} \sum_{n,r,s=1}^{\infty} (-1)^{s+r} \left\{ r \left[ \frac{e^{-m\beta(s+r)}}{m} - \beta(r+s) \text{Ei}\{-m\beta(r+s)\} \right] \right\} \\
&\quad + \frac{1}{8} \sum_{n,r,s=1}^{\infty} (-1)^r T \{ e^{-r\beta E} [f_+(s,r) \frac{\boldsymbol{\gamma} \cdot \mathbf{p}}{|\mathbf{p}|^2} - \frac{I_B I_C}{64\pi^2} \\
&\quad + h(p, \gamma) \{ f_-(n,r) \frac{I_C}{8\pi} - f_-(s,r) \} + f_+(n,r) \frac{\boldsymbol{\gamma} \cdot \mathbf{p}}{|\mathbf{p}|^2} \frac{I_B}{8\pi} ] \\
&\quad + [\{ 5 \frac{\boldsymbol{\gamma} \cdot \mathbf{p}}{|\mathbf{p}|^2} f_+(n,r) - \frac{5}{m} h(p, \gamma) f_-(n,r) \} \text{Ei}_- \\
&\quad + \frac{5E}{m^2} h(p, \gamma) f_-(n,r) \{ \frac{2e^{-rm\beta}}{m} \sinh sm\beta + \beta(r \text{Ei}_+ + s \text{Ei}_-) \} \\
&\quad + \{ \frac{2}{m} h(p, \gamma) - \frac{\boldsymbol{\gamma} \cdot \mathbf{p}}{|\mathbf{p}|^2} \} f_-(s,r) ] \} + \{ \frac{m^2}{2} \frac{\boldsymbol{\gamma} \cdot \mathbf{p}}{|\mathbf{p}|^2} + \frac{2E^2}{m} h(p, \gamma) \} \\
&\quad \times \{ \frac{2e^{-rm\beta}}{m} \sinh sm\beta + \beta(r \text{Ei}_+ - s \text{Ei}_-) \} \frac{I_C}{8\pi} ]. \tag{4}
\end{aligned}$$

From this expression for  $Z_2^{-1}$ , not only the behavior at intermediate temperatures  $T \sim m$  can be extracted but the ranges of high temperature  $T \gg m$ , low temperature  $T \ll m$ , can be also retrieved from it as limiting cases.

## 4 Results and Discussion

With the preferred order of integration of hot loops before the cold ones, the previously calculated self-mass correction terms are redone, wherever required, for all the possible ranges of temperature. This preference simplifies the calculations since the statistical effects are taken care of through hot loop momenta integrations here, before the zero temperature integration variables are dealt with. Therefore, we have re-written the electron self energy expressions in QED at the two loop level that were presented in Ref. [9]. This led to the modified expression for the relative change in the electron mass at the two loop level in Eq. (3). From these corrections one can then retrieve the results for all temperature ranges of interest here, classified as, the high temperature  $T \gg m$  (having  $m\beta \rightarrow 0$  with  $e^{-m\beta}$  falling off exponentially as compared to  $\frac{T^2}{m^2}$ ),



the low temperature  $T \ll m$  (with fermions contribution negligible) and the intermediate temperatures  $T \sim m$  (by taking  $m\beta \rightarrow 1$ ).

We calculated here, for the first time in this generalized form, the wavefunction renormalization constant using the thermal contributions to the second order self energy diagrams. The divergences get cancelled as usual by including the counter terms in Fig. 1(c) and 1(d), as already checked [9]. The fermions do not pick any contribution from the heat bath at low temperature. Therefore, the second order in  $\alpha$  corrections to the electron self energy at low temperature can be retrieved as a limiting case that contains contribution from hot photons only giving:

$$\Sigma_\beta(p) \xrightarrow{T \ll m} \frac{\alpha^2}{4\pi^3} \left[ 4\not{p} + \frac{8\pi T^2}{3m^2}(\not{p} - m) \right].$$

The wave function renormalization constant upto two loops at  $T \ll m$  from Eq. (4) is therefore

$$\begin{aligned} Z_2^{-1} \xrightarrow{T \ll m} & 1 + \frac{\alpha}{4\pi} \left( 4 - \frac{3}{\varepsilon} \right) - \frac{\alpha}{4\pi^2} \left( I_A - \frac{I^0}{E} \right) \\ & - \frac{\alpha^2}{4\pi^2} \left( 3 + \frac{1}{\varepsilon} \right) I_A + \frac{2\alpha^2 T^2}{3\pi^2 m^2}, \end{aligned} \quad (5)$$

which is the same as that in Ref. [28]. The high temperature limit for this constant gives:

$$\begin{aligned} Z_2^{-1} \xrightarrow{T \gg m} & 1 - \alpha \left[ \frac{2I_A}{\pi} + \frac{1}{4\pi} \left( \frac{3}{\varepsilon} - 4 \right) + \frac{4\pi T^2}{3} \right] \\ & - \alpha^2 \left[ \frac{1}{4\pi^3} \left\{ \frac{1}{\varepsilon} (I_A - 3J_A) + (3I_A + 5J_A) - \frac{8T^2}{3m^2} \right\} \right. \\ & + \frac{1}{8} \sum_{n,r,s=1}^{\infty} (-1)^r T \{ e^{-r\beta E} [f_+(s,r) \frac{\gamma \cdot \mathbf{p}}{|\mathbf{p}|^2} - \frac{I_B I_C}{64\pi^2} \\ & + h(p,\gamma) \{ f_-(n,r) \frac{I_C}{8\pi} - f_-(s,r) \} + f_+(n,r) \frac{\gamma \cdot \mathbf{p}}{|\mathbf{p}|^2} \frac{I_B}{8\pi} \\ & \left. + \left\{ \frac{2}{m} h(p,\gamma) - \frac{\gamma \cdot \mathbf{p}}{|\mathbf{p}|^2} \right\} f_-(s,r) \} \right]. \end{aligned} \quad (6)$$

It can be seen from Eq.(7) that the leading contribution in this range of temperature,  $T \gg m$ , at the two loop level is  $\frac{2T^2}{3m^2}$ . The two loop fermion self energy in QED has been calculated in detail recently [29] using the hard thermal loop resummation introduced by Braaten and Pisarski [30]. As far as the renormalization is concerned, the resummed hard thermal loops (HTL) do not affect it [29]. Hence  $Z_2$  does not get any contribution from HTL here. Moreover, it is worth noticing that the thermal corrections, at second order in  $\alpha$ , to the wavefunction renormalization constant at extreme temperatures (  $T \ll m$  and  $T \gg m$  ) are still proportional to  $\frac{T^2}{m^2}$  as in case of the selfmass of electron. However, the expression for the intermediate temperatures is significantly different from the self-energy expression obtained earlier [9]. The calculations around

$T \sim m$  for the mass and wavefunction renormalizations are still cumbersome at the two-loop level, but much less difficult than those in Ref. [9]. The renormalizability of the theory at finite temperature can be explicitly checked and holds through the order by order cancellation of singularities. This provides a platform to include the general effects due to chemical potential in the hot and dense background, later on. With the experience of including chemical potential at one loop level, in real time formulation [23,26,27], it is foreseen that two loop self energies will be much more complicated but is still worth-doing to develop a calculational technique for high density hot plasmas or even superfluids inside the cores of neutron stars[31]. The modified wavefunction is expected to modify the finite temperature contributions to electroweak processes [32] as well as the neutrino magnetic moments [33] up to the two loop level.

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### Figure caption

Fig. 1 Two loop electron self energy diagrams and counter terms

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